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Technical Lesson 1

MATHEMATICS

A text book on preparatory mathematics is supplied with this lesson. It is assumed that you already know how to work out simple problems in addition, subtraction, multiplication and division. Elementary principles are adequately treated throughout the different chapters in the book and this lesson is primarily intended to supplement this work.

Providing you are willing to give close attention to details and careful thought concerning different steps that are performed when computing the worked-out examples in this lesson, there is no doubt but that you will soon feel confident when handling any problems that involve the use of any instruction contained in either the text book or this lesson. Work out as many of the examples as possible that are given in the exercises in the book.

It often comes to our attention that a student will forget parts of certain mathematical operations, or perhaps he may require some practice in working out problems. Our purpose in presenting this review of mathematics at the beginning of the course is to assist those students who feel that they need help.

Mathematics may be compared to a language, since it is composed of numerous terms, expressions, and operations, all of which are founded on well established rules and laws. You should make a special effort to learn the meaning of the various terms and to thoroughly understand the rules and laws stated both in this lesson and in the book. Do not study this lesson only once and then lay it aside. Our advice to you is to frequently read through the pages and review any points which are not firmly fixed in your mind. Frequent reviews and practical application of the different principles are the only possible ways by which you will be able to cultivate your memory to retain a sufficient amount of information to be of any value to you. As a rule the principles of mathematics become very simple to most persons after they apply themselves to the task of studying the subject.

Guess work in mathematics is a very dangerous pastime and if indulged in very often it may block your progress, at least it will cause you to be careless and make mistakes and, furthermore, you are apt to waste considerable time. To get the most out of this work we advise you to learn and understand the rules first, and then practice them by working out numerous problems until they become absolutely familiar to you. It is not too much to ask of anyone who intends to educate himself above the average to work out at least forty or fifty problems relating to a particular mathematical principle. It should be remembered, however, that in order to thoroughly understand the principles involved each problem must present a slightly different appearance which may be done by changing the wording of the question and the numerical values that are used.

PART I

SQUARE ROOT DEFINED

The term "root" is best explained by means of an illustration before the definition itself is given.

Suppose we separate 4 into two equal factors, 2 and 2. This shows us that $4 = 2 \times 2$ and that one of these factors 2 is the square root of 4. By inspection it will be clearly seen that this is just the reverse or opposite process to that used when obtaining the power of a number. Let us give another illustration. Separate 16 into two equal factors, 4 and 4. Multiply 4 by itself, $4 \times 4 = 16$, or $16 = 4 \times 4$. Hence, 4 is the square root of 16.

Square root is defined as follows: The square root of any number is one of the two equal factors into which the number is divided.

THE RADICAL SIGN

To indicate a power we make use of an exponent and to indicate a root we make use of a special sign, called the radical sign. The radical sign is written $\sqrt{}$ and is followed by a vinculum, which is a straight line, drawn as an extension to the radical. In all cases the vinculum is used to show to what extent the radical sign affects the expression, or number, before which it appears.

For example, the number 16 when considered entirely by itself simply means 16 units. However, when a radical sign appears before the number 16, or $\sqrt{16}$, it means that the square root of 16 is to be taken.

In the following expression $\sqrt{16} \times 5 = 20$, the radical sign affects only 16, and not 5, since the vinculum only extends over 16.

A small number placed in the $\sqrt{}$ shaped opening of a radical is called the INDEX and indicates what root is to be taken. Thus, $\sqrt[3]{125}$ would tell us at once that the cube root of 125 is to be found, and $\sqrt[4]{192}$ indicates that the fourth root of 192 is to be found. Inasmuch as square root is more frequently used in ordinary computations it is the custom in this case to omit the figure 2 from the radical. Therefore, to indicate square root, a radical with its associated vinculum is written $\sqrt{}$ without an index number.

For example, the square root of 16 is always written as $\sqrt{16}$, and never as $\sqrt[2]{16}$. However, bear in mind that if any root higher than 2 is desired the index must always be written.

Summing up all of the facts in the foregoing paragraphs we find that "Involution" is used when the power of a certain number is to be found whereas, "Evolution" is used when the root of a certain number is to be found.

SQUARE ROOT

There are only a very few numbers that are perfect squares. For example, between 1 and 100 there are only 9 perfect squares, and between 1 and 1000 there are but 31 perfect squares.

The following list includes all of the numbers between 1 and 1000 that are PERFECT SQUARES: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484, 529, 576, 625, 676, 729, 784, 841, 900, 961.

If we extract the square root of any of the above numbers we will obtain a result which is a whole number.

HOW TO FIND THE SQUARE ROOT OF A PERFECT SQUARE

EXAMPLE: Find the square root of 625.

$$\sqrt{625}$$

First Step: Write down the statement of the problem, as shown above, by using the radical sign. Next separate the number 625 into periods of two digits each as indicated below where the radical sign is omitted when working out the problem. Begin at the right of the number as follows and place a small mark between each pair of figures or digits to indicate this separation. (Note: The term "digit" and "figure" are used here interchangeably.) The small mark or stroke is written thus:

6'25

It is found that when 625 is separated in this way it contains two periods and from this we know that only two figures will appear in the root, or answer.

6'25

Second Step: Now select the largest figure, which when squared (that is, the figure selected is multiplied by itself) will be equal to or less than the left hand period, or 6. Suppose we try 1. Squaring 1 we have $1 \times 1 = 1$; this result is obviously too small. Let us try 3. Squaring 3 we have $3 \times 3 = 9$. But, we also find 9 to be too large, for it is greater than the left hand period. Try 2 this time. Squaring 2 we have $2 \times 2 = 4$. We find that 2 is the correct figure to use, according to our rule, because 2 squared, or 4, is the only number below 6 which when squared will not be greater than 6.

Third Step: Place 4 under 6, and then place the square root of 4, or 2, in the bracket to the right which is drawn expressly for this purpose, as follows:

$$\begin{array}{r} 6'25 \mid 2 \\ \underline{4} \\ 2 \end{array}$$

Fourth Step: Subtract 4 from 6 and write down the remainder as you would do in any regular process of subtraction, thus:

$$\begin{array}{r} 6'25 \mid 2 \\ \underline{4} \\ 2 \end{array}$$

Fifth Step: Bring down the next period of the number, or 25. Our computation will now look as follows:

$$\begin{array}{r} 6 \overline{) 25} \quad | 2 \\ \underline{4} \\ 225 \end{array}$$

Sixth Step: Now multiply the root 2, just found, by 2. Write down this work at one side of the paper, as follows:

$$2 \times 2 = 4$$

$$\begin{array}{r} 6 \overline{) 25} \quad | 2 \\ \underline{4} \\ 225 \end{array}$$

Seventh Step: The figure 4, just found by multiplying the root (2) by 2, is called the trial divisor. We must now divide the remainder 225 by 4. Remember, that at this point you always disregard the right hand figure in the remainder which is 5 in this example. Actually, then, we divided 22 by 4, and not 225 by 4. Hence, $22 \div 4 = 5$. We find that we have a quotient 5 with a remainder. Disregard the remainder but use the quotient 5, and annex it to the 4 which was found in the 6th step by doubling the root. Our example and work will appear as follows:

$$2 \times 2 = 4 \text{ annex 5 makes } 45$$

$$\begin{array}{r} 6 \overline{) 25} \quad | 2 \\ \underline{4} \\ 225 \end{array}$$

Eighth Step: The number 45 just found is called the true or complete divisor. The steps explained above are now repeated. Accordingly, we divide 225 by 45. By simple division we find that $225 \div 45 = 5$. This figure 5 becomes the second digit in the root and, therefore, it is written in the bracket to the right of the first figure in the root, or 2. The root now contains two figures, or digits, 2 and 5.

$$\begin{array}{r} 6 \overline{) 25} \quad | 25 \\ \underline{4} \\ 225 \\ \underline{225} \end{array}$$

This completes the necessary steps in the working out of our example, and we find that 25 is the square root of 625. Since there is no remainder we call this a perfect square.

Proof: To check our computation and prove that 25 is the correct root we merely have to multiply 25 by itself, that is, we square 25. If the number itself and the root, when squared, are equal then you know that you have the correct answer. Thus, $25^2 = 25 \times 25 = 625$.

PART II

HOW TO FIND THE SQUARE ROOT OF A LARGE NUMBER. The examples in square root that have been previously worked out consisted of numbers having only three digits, all of the numbers being perfect squares. This section will also deal with perfect squares, but, this time we will work with numbers consisting of more than three digits so that the square root of any large number may be found. Study the following worked out example.

EXAMPLE: Find the square root of 516961.

Mark off the periods to include two digits each, thus:

51'69'61 |

The number whose square is equal to, or less than the first period, 51, is found by trial to be 7.

$$7 \times 7 = 49$$

Since 7 is to be the first figure in the root, it is placed in the bracket to the right of the number, as shown below:

51'69'61 | 7

Square the root 7, just found (that is, multiply 7 by itself, or 7×7) and place the result, 49, under the first period of the number, 51, and then subtract 49 from 51.

$$7 \times 7 = 49$$

51'69'61 | 7
49
—
2

After subtracting, we get a remainder of 2, and to this we annex the next period of the number, 69, thus giving us a remainder of 269.

$$7 \times 7 = 49$$

51'69'61 | 7
49
—
269

Multiplying the root 7 by 2 gives us a trial divisor of 14.

$$7 \times 7 = 49$$

$$7 \times 2 = 14$$

51'69'61 | 7
49
—
269

By division we find that the trial divisor 14 will go into the remainder 269 only once. Note that we disregard the right hand figure 9 in 269, and, therefore, we really divide 26 by 14, as follows:

$$26 \div 14 = 1 +$$

Annex the 1 to the trial divisor 14, thus making 141 the complete divisor.

$$7 \times 7 = 49$$

$$7 \times 2 = 14 \text{ annex } 1 = 141$$

51'69'61 | 7
49
—
269

Next divide 269 by the complete divisor 141 and use the result as the next figure of the root.

$$269 \div 141 = 1 +$$

We find 1 to be the next figure of the root, and, therefore, 1 is written to the right of 7. Now bring down 69 as in any ordinary division. These steps are shown below.

$$\begin{aligned} 7 \times 7 &= 49 \\ 7 \times 2 &= 14 \text{ annex } 1 = 141 \end{aligned}$$

$$\begin{array}{r} 51'69'61 \mid 71 \\ \underline{49} \\ 269 \\ \underline{141} \\ 128 \end{array}$$

Multiply 1 by 141 = 141. Write 141 under 269 and then subtract. This leaves a remainder of 128. The next period of the number, or 61, is now brought down and annexed to 128.

$$\begin{aligned} 7 \times 7 &= 49 \\ 7 \times 2 &= 14 \text{ annex } 1 = 141 \end{aligned}$$

$$\begin{array}{r} 51'69'61 \mid 71 \\ \underline{49} \\ 269 \\ \underline{141} \\ 12861 \end{array}$$

Multiply the entire root 71 by 2 in order to obtain a product that will be the next trial divisor. This product is 142.

$$\begin{aligned} 7 \times 7 &= 49 \\ 7 \times 2 &= 14 \text{ annex } 1 = 141 \\ 71 \times 2 &= 142 \end{aligned}$$

$$\begin{array}{r} 51'69'61 \mid 71 \\ \underline{49} \\ 269 \\ \underline{141} \\ 12861 \end{array}$$

Now divide 12861 by 142, but first disregard the extreme right hand digit 1 of 12861. In reality, then, we divide 1286 by 142, or

$$1286 \div 142 = 9$$

After annexing the 9 to 142, we get 1429, the complete divisor.

$$\begin{aligned} 7 \times 7 &= 49 \\ 7 \times 2 &= 14 \text{ annex } 1 = 141 \\ 71 \times 2 &= 142 \text{ annex } 9 = 1429 \end{aligned}$$

$$\begin{array}{r} 51'69'61 \mid 71 \\ \underline{49} \\ 269 \\ \underline{141} \\ 12861 \end{array}$$

Next divide 12861 by the complete divisor 1429, or

$$12861 \div 1429 = 9$$

By division it is found that 1429 will go into 12861, 9 times, as indicated above. This is how the next digit, 9, of the root is obtained. We then set down 9 to the right of digits, 7 and 1, which we previously found, thus giving us the total root 719.

$$\begin{array}{l}
 7 \times 7 = 49 \\
 7 \times 2 = 14 \text{ annex } 1 = 141 \\
 71 \times 2 = 142 \text{ annex } 9 = 1429
 \end{array}$$

$$\begin{array}{r}
 51'69'61 \overline{) 719} \\
 \underline{49} \\
 269 \\
 \underline{141} \\
 12861
 \end{array}$$

Multiply the complete divisor 1429 by the last root found, or 9.

$$1429 \times 9 = 12861$$

Write the product 12861 under the remainder 12861.

$$\begin{array}{l}
 7 \times 7 = 49 \\
 7 \times 2 = 14 \text{ annex } 1 = 141 \\
 71 \times 2 = 142 \text{ annex } 9 = 1429
 \end{array}$$

$$\begin{array}{r}
 51'69'61 \overline{) 719} \\
 \underline{49} \\
 269 \\
 \underline{141} \\
 12861 \\
 \underline{12861} \\
 0
 \end{array}$$

The example finally works out with even values and we obtain an answer which is a whole number. Since there is no remainder 719 is the complete root. That is, 719 is the square root of the number 516961.

Proof: To prove the answer we must square the complete root 719. If the product of 719 x 719 equals the original number, or 516961, then you are assured that you have worked the problem correctly. The computation below proves that 719 is correct because 719² or 719 x 719 equals 516961.

$$\begin{array}{r}
 719 \\
 \underline{719} \\
 6471 \\
 \underline{719} \\
 5033 \\
 \underline{516961}
 \end{array}$$

A TEST EXERCISE IN SQUARE ROOT:

Find $\sqrt{2951314276}$.

The solution of a typical example in square root is given below. The student should work out this exercise independently and check each step with the original explanations for the purpose of testing his ability to perform all of the necessary steps correctly.

$$\begin{array}{r}
 29'51'31'42'76 \overline{) 54326} \\
 \underline{25} \\
 451 \\
 \underline{416} \\
 3531 \\
 \underline{3249} \\
 28242 \\
 \underline{21724} \\
 651876 \\
 \underline{651876} \\
 0
 \end{array}$$

PART III

HOW TO FIND THE SQUARE ROOT OF AN INTEGRAL NUMBER.

To determine the square root of an Integral number which is not a perfect square, the student should proceed according to the outline in Part II with exceptions as stated below.

When solving for the square root of an integral number, which is not a perfect square, first determine how many digits, or figures, are required in the answer, and then annex additional ciphers to the right of the number until there are as many decimal periods provided as there will be decimal places (meaning digits) appearing in the root.

Once this has been determined the extracting of the root is done exactly as explained in Part II.

The student must keep in mind that the exact root of a number, which is not a perfect square, can not be found. An example of this kind is worked out in the paragraphs immediately following.

EXAMPLE: Find: $\sqrt{39}$

The first thing that one must do is to decide upon the number of decimal places (digits) desired in the answer. Suppose you wish to work out the above example to four decimal places, then write down number 39 and annex four decimal periods to the right, consisting of two ciphers each, as shown below:

39.00'00'00'00

The example is now in correct form for solution. The square root is extracted by applying the principles and step-by-step procedure already given in the foregoing parts of this lesson. The first thing to do is to select a figure which when squared will be closest to 39 but not greater than it. This figure is 6, and is written down as the first figure in the root, as follows:

$$6^2 = 36$$

$$6 \times 2 = 12 \text{ annex 2 makes } 122 \text{ complete divisor}$$

$$62 \times 2 = 124 \text{ annex 4 makes } 1244 \text{ complete divisor}$$

$$624 \times 2 = 1248 \text{ annex 4 makes } 12484 \text{ complete divisor}$$

$$6244 \times 2 = 12488 \text{ annex 9 makes } 124889 \text{ complete divisor}$$

$$\begin{array}{r} 39.00'00'00'00 \quad \underline{6.2449 +} \\ 36 \\ \hline 300 \\ 244 \\ \hline 5600 \\ 4976 \\ \hline 62400 \\ 49936 \\ \hline 1246400 \\ 1124001 \\ \hline 122399 \end{array}$$

The decimal point is easily and quickly located in the answer by counting off the same number of decimal places (digits) to the right of the answer, beginning at the decimal point, as there are decimal periods in the example.

Since they were four decimal periods annexed to the right of the decimal point, following the number 39, then the answer will contain four decimal places as indicated above.

Proof: To check the correctness of the above problem we must square the answer, that is, 6.2449 must be multiplied by itself. If you perform this multiplication you will find that the product will not equal the number 39, but will very closely approach 39. This proves that the exact root of an imperfect square can not be determined.

PART IV

GENERAL RULES

The rules given in Parts I, II and III for extracting the square root of a number were applied first to integral numbers which were perfect squares, and second, to integral numbers which were imperfect squares.

The next part of our work involves only fractional numbers. Before proceeding further, however, we will state several rules that may be applied to any example in square root involving perfect squares, imperfect squares, or fractional numbers. This is more or less of a review of what has been presented to you in this lesson thus far. The principles which you have already learned may be applied to examples where the square root of fractional numbers must be found.

RULE 1

(A) ---BEGIN AT THE DECIMAL POINT and divide the number, whose square root is to be found, into periods of two digits each.

When the integral part of the number contains an odd number of digits then only one digit will appear in the left-hand period. This period may contain one or two digits as the case may be. However, in the case of a decimal proceed as explained in the following paragraph (B).

(B) ---When the decimal part of the number contains an odd number of digits annex a cipher at the right, so that the last period of the decimal will contain two digits.

Explanation of Rule 1.

(A) ---Example:

(Decimal point)									
Integral					Decimal				
part					part				
5	5	4	3	2	.	6	4	1	

Separate, or point off, the number into periods of 2 digits each, beginning at the decimal point, as follows:

<u>Point off to left</u>	<u>Point off to right</u>
5'54'32	.64'1

- (B) — Since the integral part contains an odd number of digits the extreme left-hand period contains only one digit. As stated before this period will have one or two digits according to the total number of digits comprising the integral part of the number.

Since the decimal part of the number consists of an odd number of digits a cipher must be added to complete the last period on the right, thus:

5'54'32.64'10

One digit in left hand period of integral number.

Cipher is annexed to complete right-hand period of decimal part of number.

RULE 2.

- (A) — A certain number is selected which when squared will be equal to or less than the first period on the left hand. This number, called a potential root, is squared and the result is placed under the first period.
- (B) — The number, itself (not squared), which was selected according to (A) above will be the first digit in the required root.
- (C) — The number, when squared, is subtracted from the first period and the remainder is written down.
- (D) — After you subtract the greatest square from the first period, as suggested in (C), bring down the next period and annex it to the remainder found.

Explanation of Rule 2.

- (A) — To illustrate Rule 2 consider number 55432.641 whose square root is to be found.

Separate the number into periods of two digits each and annex a cipher to the right-hand period of the decimal part,

5'54'32.64'10 | _____

Remember that we are now explaining how to begin your computation and what to do with the first period. Let us try 3 as the first digit in the root. Now square 3, which equals 9. Since 9 is greater than the left-hand period 5, we must try some smaller figure.

This time let us try 1. Now square 1, which equals 1. It is clear that figure 1 is too small.

Try 2. Now square 2 which equals 4. Figure 2 is correct, inasmuch as 1^2 is too small and 3^2 is too large.

- (B) — Therefore, 2 becomes the first digit of the root, and is written in the bracket as follows:

5'54'32.64'10 | 2 _____

- (C) — Subtract the square of the root 2, which is 4, from the first period 5 of the number. This gives us a remainder of 1.

$$2^2 = 4$$

$$\begin{array}{r} 5'54'32.64'10 \quad | 2 \\ \underline{4} \\ (\text{remainder}) \quad 1 \end{array}$$

- (D) — Bring down the next period of the number, or 54, and annex it to remainder 1 which makes the total remainder 154.

$$\begin{array}{r} 5'54'32.64'10 \quad | 2 \\ \underline{4} \\ (\text{remainder}) \quad 154 \end{array}$$

RULE 3.

- Rule 3 explains how to find the second digit in the root.
- (A) — Now double the root already found, or in other words, multiply the root by 2, and use the product as a trial divisor for the total remainder.
- (B) — Determine how many times the trial divisor, or the product obtained by doubling the root, is contained in the total remainder disregarding the right hand figure as we always do in this operation. This will be the next digit of the root and it should be placed in the bracket to the right of the first digit, and also annexed to the right of the trial divisor to make a complete divisor.
- (C) — Multiply the complete divisor by the last digit placed in the root and write this product under the total remainder.
- (D) — Perform the necessary subtraction and then bring down the next period, annexing it to the remainder just found by subtraction. Now proceed with the problem by repeating this procedure for each digit in the root.

Explanation of Rule 3.

- (A) — Double the root just found, and use the product as the trial divisor.

$$\begin{array}{l} \text{Doubling the root} \\ 2 \times 2 = 4 \end{array}$$

$$\begin{array}{r} 5'54'32.64'10 \quad | 2 \\ \underline{4} \\ 154 \end{array}$$

Double this root. The work is done at the left hand side of the problem.

Since root 2, when doubled equals 4, then 4 becomes the trial divisor.

- (B) — Find how many times the trial divisor 4 will go into the total remainder 154, disregarding the right-hand figure. This is, in reality, dividing 15 by 4, or $15 \div 4 = 3$.

The result of this last division, or dividing 15 by 4, will give us 3, which becomes the next digit in the root, and by annexing 3 to the trial divisor 4 we obtain the complete divisor, or 43.

Trial divisor.

$2 \times 2 = 4$ annex 3 makes 43

$$\begin{array}{r} 5'54'32.64'10 \quad | \quad 23 \\ \underline{4} \\ 154 \end{array}$$

Digit 3 is now written to the right of digit 2, which was previously found, and the root thus far is 23.

Annexing 3 to the trial divisor makes 43 the complete divisor, as shown above at the left-hand side.

- (C) — Multiply the complete divisor 43, just found, by the last digit of the root, or 3, and place the product, or 129, under the total remainder 154.

$2 \times 2 = 4$ annex 3 makes 43

$$\begin{array}{r} 5'54'32.64'10 \quad | \quad 23 \\ \underline{4} \\ 154 \\ \underline{129} \end{array}$$

- (D) — The product of 43×3 , or 129, is subtracted from 154 and the next period 32 of the number is brought down as follows:

$$\begin{array}{r} 5'54'32.64'10 \quad | \quad 23 \\ \underline{4} \\ 154 \\ \underline{129} \end{array}$$

2532 ← Period 32 brought down
and added to 25 gives
2532, the total remainder.

If we continue our computation to two decimal places according to the same rules we will obtain the following result:

$$\begin{array}{r} 5'54'32.64'10 \quad | \quad 235.44 + \\ \underline{4} \\ 154 \\ \underline{129} \\ 2532 \\ \underline{2325} \\ 20764 \\ \underline{18816} \\ 194810 \\ \underline{188336} \\ 6474 \end{array}$$

RULE 4

After a problem has been worked out to the desired number of decimal places then point off as many (decimal) digits in the root, beginning at the extreme right, as there are decimal periods in the number. There are two decimal periods in the above number, therefore, there will be two decimal places in the answer, as indicated.

Explanation of Rule 4.

After completing the foregoing computations and applying the rule about locating the decimal point we find that the square root of 55432.641 is 235.44 +. The + sign indicates that the answer is not complete, but is correct to two decimal places. Also, the + sign signifies that the real answer, which would be obtained if computations were carried beyond this point, would be greater than the answer which we accept as sufficiently accurate.

RULE 5

The following rule applies only in special cases.

When a trial divisor is too large to go into a remainder, then a cipher should be placed in the root and a cipher also annexed to the trial divisor. Then bring down the next period of the number and carry out the computation in the regular manner.

The following example is given to illustrate how this rule is applied.

EXAMPLE: Find: $\sqrt{41653}$

$$\begin{array}{r} 2 \times 2 = 4 \text{ trial divisor} \quad 4'16'53 \quad | \quad 2 \\ \underline{4} \\ 16 \end{array}$$

By inspection it is found that trial divisor 4 can not be divided into the remainder 16 after we disregard the right hand figure 6. That is, 4 will not go into 1.

Therefore, place a cipher in the root and also annex a cipher to the trial divisor 4 making a new trial divisor 40. Then bring down the next period of the number 53 and continue with the problem.

2 x 2 = 4 annex 0 makes
40 the new trial divisor.

$$\begin{array}{r} 4'16'53 \quad | \quad 204 \\ \underline{4} \\ 1653 \end{array}$$

Disregarding the right hand figure in the remainder 1653 gives us 165, and dividing 165 by 40, we obtain 4. Therefore, the next digit in the root is 4.

Annex 4 to the final trial divisor 40, making it 404. Multiply 404 by 4, the last digit of the root, and place the product under the remainder 1653 and subtract.

$2 \times 2 = 4$ annex 0 makes
40 the new trial divisor.

$$404 \times 4 = 1616$$

$$\begin{array}{r} 4 \overline{) 41'16'53} \quad | \quad 204 \quad + \\ \underline{4} \\ 1653 \\ \underline{1616} \\ 37 \end{array}$$

Thus, we find that the square root of 41,653 is 204 with a remainder.

PART V.

HOW TO FIND THE SQUARE ROOT OF A DECIMAL.

To find the square root of a decimal or fraction is just as simple as when using integral numbers or whole numbers. Decimals might appear more difficult to handle at first because the average student does not ordinarily work with them as frequently as with whole numbers.

EXAMPLE: Find $\sqrt{0.00025}$

A typical example with full explanations is given below. Keep in mind that the first thing to do in any case of this kind is to point off the number into periods of two digits each beginning at the decimal point, thus:

.00'02'5

After dividing the number into periods, as shown above, we notice that the right-hand period contains but one digit, 5, and therefore a cipher must be annexed to complete this period.

.00'02'50

The computation is now started by selecting a number which is the trial divisor. This number when squared must not be greater than the value of the first period. In this particular instance the first period is 00 and it is quite obvious that there is no number which is equal to zero when squared, hence, a cipher is placed in the bracket as the first digit in the root.

$$0 \times 0 = 0$$

$$\begin{array}{r} .00'02'50 \quad | \quad .0 \\ \underline{0} \end{array}$$

Bring down the next period, 02, and continue with the example as explained in our instructions which preceded this part. A number is now selected which when squared will not be greater than the value of the second period, or 02. The number which will satisfy this condition is found to be 1, hence, 1 is written in the bracket as the next digit in the root.

$$0 \times 0 = 0$$

$$1^2 \text{ or } 1 \times 1 = 1$$

$$\begin{array}{r} .00'02'50 \quad | .01 \\ \underline{0} \\ 02 \end{array}$$

After figure 1 is written in the bracket as just suggested, also place 1 under the remainder 2, and then subtract.

$$0 \times 0 = 0$$

$$1 \times 1 = 1$$

$$\begin{array}{r} .00'02'50 \quad | .01 \\ \underline{0} \\ 02 \\ \underline{1} \\ 1 \end{array}$$

This leaves a remainder of 1 to which the next period 50 is annexed. The total remainder is now 150.

$$0 \times 0 = 0$$

$$1 \times 1 = 1$$

$$\begin{array}{r} .00'02'50 \quad | .01 \\ \underline{0} \\ 02 \\ \underline{1} \\ 150 \end{array}$$

Multiply the root 1 (which is written in the bracket) by 2 in order to obtain a trial divisor and then determine the number of times this trial divisor, or 2, is contained into 150, disregarding the right-hand digit in 150. This means, of course, that 15 is to be divided by 2, which equals 7. This figure just obtained, or 7, is called the potential root. We now annex 7 to the trial divisor 2 giving us 27 and by multiplying 27 by the potential root 7 we obtain the figure 189.

$$27 \times 7 = 189$$

Since figure 189 is larger than the total remainder 150 it is very evident that we cannot use 7 as the next digit in the root. Therefore, we must try a number which is less than 7. Let us try 6. The figure 6 now becomes the potential root.

The figure 6 or potential root, is annexed to the trial divisor 2 and we have 26. Multiplying the potential root 6 by 26 equals 156. We cannot use 6 as the next digit in the root since 156 is greater than 150, and therefore, we must try a smaller figure. Let us try 5 this time.

Apply the same line of reasoning in regard to the use of 5 as we did in the case of the other trial divisors, 7 and 6, which could not be used.

Trial divisor 2 with 5 annexed equals 25. Potential root 5 multiplied by 25 equals 125. Since 125 is not greater than 150 then it is evident that 5 can be applied as the next digit in the root.

Therefore, write 5 in the bracket as the next digit in the root; then multiply the final divisor, or 25, by the root 5, and place the total, or 125, under 150.

$$0 \times 0 = 0$$

$$1 \times 1 = 1$$

$$1 \times 2 = 2 \text{ annex 5 makes } 25 \text{ complete divisor}$$

$$25 \times 5 = 125.$$

$$\begin{array}{r} .00'02'50 \quad | .015 + \\ \underline{0} \\ 02 \\ \underline{1} \\ 150 \\ \underline{125} \\ 25 \end{array}$$

If the answer is sufficiently accurate, for the purpose at hand, when the root is carried out to three decimal places, then our work is completed. But, suppose an answer containing four decimal places is required; then we must go through one more computation according to the principles already outlined.

To give you additional practice in working out examples of this kind let us carry out the answer to one more decimal place. To do this annex a period consisting of two ciphers to the right of the last period 50 and continue with the work.

$$0 \times 0 = 0$$

$$1 \times 1 = 1$$

$$1 \times 2 = 2 \text{ annex 5 makes } 25$$

$$25 \times 5 = 125$$

$$15 \times 2 = 30 \text{ annex 8 makes } 308 \text{ complete divisor}$$

$$308 \times 8 = 2464$$

$$\begin{array}{r} .00'02'50'00 \quad | .0158 + \\ \underline{0} \\ 02 \\ \underline{1} \\ 150 \\ \underline{125} \\ 2500 \\ \underline{2464} \\ 36 \end{array}$$

Hence, we have found that the square root of 0.00025 is 0.0158 + . This answer is correct to four decimal places.

EXAMPLE: Given the number .0005 find its square root. The answer should contain 4 decimal places.

The solution to this example is given below without all of the work being shown. This is done in order to give you an opportunity to work the same example step-by-step, showing all of the work, and afterward to check your computation.

$$\begin{array}{r} .00'05'00'00 \quad | .0223 + \text{ Answer.} \\ \underline{0} \\ 05 \\ \underline{4} \\ 100 \\ \underline{84} \\ 1600 \\ \underline{1329} \\ 271 \end{array}$$

HOW TO FIND THE SQUARE ROOT OF A FRACTION.

EXAMPLE: Find: $\sqrt{\frac{25}{81}}$

In this problem it is seen that both the numerator, or 25, and denominator, or 81, are perfect squares. Therefore, to work out this problem we simply take the square root of the numerator and denominator separately and then remove the radical sign as follows:

The square root of 25 is 5; and that of 81 is 9. Hence, $\sqrt{\frac{25}{81}} = \frac{5}{9}$

Simple examples like the one given above can be solved merely by inspection. However, this cannot be done when the numerator and denominator are not perfect squares. In an example of the latter kind you must first reduce the fraction to a decimal and then proceed to find the square root of the decimal.

EXAMPLE: Find: $\sqrt{\frac{2}{7}}$

First reduce the fraction $\frac{2}{7}$ to a decimal by dividing 2 by 7. Thus, we find that $\frac{2}{7} = 0.28571 +$.

The next step is to find the square root of 0.28571. The same step-by-step procedure is followed in all cases where a decimal is involved in square root.

After working out this problem you should obtain 0.5345 for the answer which is not fully complete since there is a remainder, but, in the majority of cases a decimal carried out to three or four places is accepted as sufficiently accurate for all practical purposes. In expressing this result we would say that 0.5345 is the square root of $\frac{2}{7}$ correct to four decimal places.

EXAMINATION - LESSON I

Your answers to Problems 1, 2, 3, and 4 should be given in four decimal places. DO NOT show the work in these problems; only the answers are required.

1. What is the square root of 527849?
2. What is the square root of 0.94935?
3. What is the square root of 26.842?
4. What is the square root of $\frac{144}{255}$?

Your answers to Problems 5, 6, 7, 8, 9, and 10 should be accompanied by all of the work necessary to find the answers.

5. Add: $\frac{1}{2} + \frac{2}{3} + \frac{4}{7}$
6. Multiply: $\frac{5}{8} \times \frac{11}{16}$ and reduce the product found to a decimal.
7. Divide: $\frac{14}{17} \div \frac{28}{32}$
8. Multiply: 1.005×0.175
9. Divide: $\frac{4785.85}{15.3}$
10. Reduce the fraction $\frac{18}{35}$ to a decimal.